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Abstract. Possible effects on the liner implosion of an early plasma formation outside the liner are discussed. At the modest density and temperature this plasma is a sufficiently good conductor to trap the pre-imposed axial magnetic field. The rising axial current compresses the plasma and axial field towards the liner surface and creates high-magnetic-field filamentary structures that seed the perturbations by the mechanism proposed in Ref. 1, but in a significantly higher field. Possible sources of this plasma are briefly discussed.

Recent experiments on the liner implosion in the presence of a pre-imposed axial magnetic field [1] have revealed the emergence of robust helical structures on the outer surface of the liner. In principle, the simultaneous presence of both axial and azimuthal field can be expected to produce such structures [2] via the magnetohydrodynamic (MHD) instabilities, but the axial field used in the experiment seemed to be too weak to explain quite a noticeable pitch-angle observed experimentally. To explain this paradoxical result, the authors of Ref. 1 have proposed several plausible mechanisms. In this note we discuss one of such mechanisms: entrainment and compression of the weak axial field by a plasma formed outside the liner early in the pulse. The helical structures in this model appear in the total magnetic field: the field of the high pinch current and of the enhanced axial field.

This note contains just a general outline of the key assumptions and identifies a few interesting details of this model. Much more effort would be needed in order to be able either to accept or reject the model.

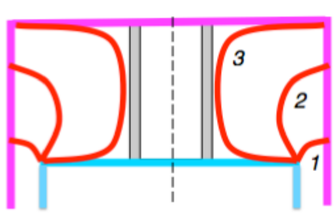

Fig. 1 Schematic of the early stage of the discharge. Cathode is shown in blue and anode in magenta. The liner (that has not started moving yet at this stage) is shown in grey. Three consecutive positions of the conducting shell that is pushed inward by the pinch force are shown in red.

The mechanism of sweeping of the pre-imposed axial field towards the liner surface is illustrated in a cartoon form in Fig. 1. Assume that early in the pulse, well before the current approached its maximum value, an axisymmetric plasma “bridge” is formed (for example, due to the loss front that precedes magnetic insulation) near the anode-cathode gap through which the power is supplied, position “1” in Fig. 1. If the plasma in this bridge is highly conducting (see estimates below), the axial (poloidal) magnetic flux enclosed by it will remain constant. On the other hand, the increasing magnetic pressure of the azimuthal (toroidal) magnetic field would push the conducting shell towards the axis and thereby compress the trapped axial flux near the liner surface. The successive shapes 1-3 of the plasma shell are qualitatively accounting for the possible frozen-in conditions for the axial field on the electrodes, whence a strong deviation from the cylindrical shape.

In the numerical estimates below we use the following set of parameters: Liner outer radius a=0.35 cm, liner thickness 0.05 cm, liner height l=0.8 cm, radius of the return current can b=1cm. The current at this early stage of the implosion will be approximated by a linear function of time [3], $I=I_0(t/\tau)$, with I_0 = 10 MA, and τ = 50 ns. Initial axial magnetic field is assumed to be B_0=10 T. These are some generic parameters, not referring to any particular shot, but suitable for the order-of-magnitude estimates.

In the ideal case of no axial flux leakage, this flux could be compressed to a narrow annulus near the liner surface. The minimum possible thickness of this high-field layer is determined by the skin depth in the liner material. For rough estimates, we assume that the magnetic diffusion coefficient D_m is ~ 300 cm^2/s, and characteristic time is ~ 50 ns. Then, the thickness of the skin layer δ_{skin} is ~ 0.006 cm. This then yields the following estimate for the absolute maximum of the axial field near the liner surface: $B \sim B_0(\pi b^2)/(2\pi a \delta_{skin}) \sim 250\ B_0$ (!). This estimate, being unrealistic for a variety of reasons (see below), still shows that there is a large reservoir of the axial flux in the setting shown in Fig. 1.

More realistic limits on the maximum axial field come out from the assessment of the current-carrying capability of the plasma shell. Assuming that the thickness of the plasma shell is $\delta_{shell,}$ and the distance of the shell from the axis is comparable to the radius of the liner a, we find that the relative velocity of the electrons and the ions u_z is related to plasma current by the following equation:

$$u_z(cm/s) \approx \frac{10^{24} I_z(MA)}{a(cm)\delta_{shell}(cm)n_e(cm^{-3})} \tag{1}$$

At the same location the azimuthal magnetic field and the corresponding Alfven velocity will be

$$B_\varphi(MG) = \frac{0.2 I_z(MA)}{a(cm)};\ \ \mathrm{v}_A = \frac{2.5\times 10^{17} B_\varphi(MG)}{\sqrt{(A/Z)n_e(cm^{-3})}}\,, \tag{2}$$

where A and Z are the atomic mass and charge state of the plasma ion. Significance of the relation between the current velocity u_z and the Alfven velocity v_A is related to the onset of the Hall effect and transition to a so called Hall MHD or Electron MHD, where the current could be significantly inhibited by a variety of processes [4]. In the same direction act various electrostatic instabilities that may lead to the effect of anomalous resistance (see review [5]). They appear when the "current" velocity exceeds a few sound speeds c_s. Figure 2 contains the corresponding plots for the shell thickness of 0.2 cm. Even for this rather large shell thickness the plasma density required to avoid the appearance of the anomalous resistivity is high, in the range of 10^{17} cm^{-3}. The plots corresponds to the partially ionized carbon plasma with Z=3. One sees from this example that it is hard to expect that the magnetic flux would be compressed to a layer thinner than ~ 0.2 cm. Then, the same as before estimate of the compressed magnetic field yields an enhancement of ~ 10, i.e., to ~100 T for the 10 T bias field. This can happen when the azimuthal field reaches the value higher than that, i.e., at the pinch current exceeding ~ 2 MA.

In the absence of the anomalous resistance, the magnetic field penetration through the plasma shell will be determined by the resistive magnetic diffusivity. Using the estimate $D_M(cm^2/s) \sim 4\times 10^6 Z/[T_e(eV)]^{3/2}$ with Z=3, one finds that the diffusion time through a 2 mm thick shell will be ~4 μs, i.e., much longer than the characteristic time τ.

A strong magnetic shear present in the current sheath may lead to a reconnection process [6] that would cause "mixing" of the axial and azimuthal field, so that actually the field that would be pushed towards the liner surface will have a helical structure. Characteristic time for the reconnection will be a few Alfvenic times and, for the Alfven velocity as in Eq. (2) with I_z=2-5 MA, it will be ~ a few ns. In other words, the field "swept" to the liner surface will have a helical structure, with the current flowing predominantly along the field lines. The latter conclusion stems from the fact that the anticipated plasma pressure will be much less than the magnetic pressure (the parameter $\beta = p_{plasma} / p_{magn}$ where p_{plasma} and p_{magn} are the plasma and magnetic pressure, respectively, is for n_e=10^{17} cm^{-3}, T=100 eV, and B=100 T only 4×10^{-4}), so that the instantaneous equilibrium must be force-free, with the current density $\boldsymbol{j}$ parallel to $\boldsymbol{B}$. If the azimuthal field near the liner surface is higher than the axial field, the current is predominantly azimuthal. The corresponding electron velocity is related to the axial field by equation similar to Eqs. (1), (2): $u_\varphi(cm/s) = 5\cdot10^{24} B_z(MG)/\delta_{shell}(cm)n_e(cm^{-3})$. For B_z~100 T and other parameters as before, the velocity u_φ is 10^8 cm/s and is smaller than both electron thermal velocity and Alfven velocity.

Note that the current will be subject to tearing instability that will tend to split it into several helical filaments aligned with the (helical) magnetic field [6]. In other words, the current in the vicinity of the liner surface will have a structure of several helical filaments. Note also that a significant fraction of the axial current will gradually switch to the liner surface, as in a standard "vacuum" Z pinch.

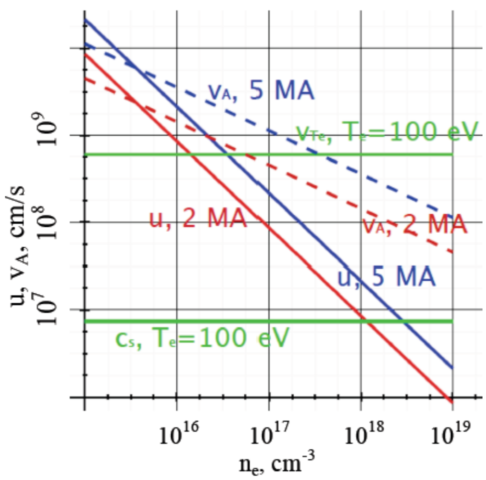

Fig. 2 Towards the current-carrying capability of the plasma shell. Solid lines, blue and red, correspond to the current velocity (Eq. (1)) for two currents, 5 and 2 MA, and δ_{shell}=0.2 cm. Dashed lines characterize the Alfven velocities. The current inhibition may take place above the intersection of the curves corresponding to the same current. Upper green line corresponds to the electron thermal velocity, and the lower green line, to the ion sound speed. Development of the ion acoustic instability (see review [5]) that would appear at u exceeding a few c_s is a possibility for the 5MA case. All estimates are made for Carbon in the charge state Z=3.

The presence of these helical current structures near the surface of the liner will inevitably produce the image currents of the same structure flowing on the liner surface. These currents would then cause various types of helical perturbations within the skin layer. In particular, they would produce helical pattern of increased temperature and decreased density, as well as the corresponding pattern of the increased resistivity. These perturbations would then seed a hierarchy of instabilities, from electrothermal instability [7] to MRT instability [1]. Analysis of the further development of these perturbations goes well beyond the scope of this qualitative discussion. We limit ourselves to noting that the anticipated patterned temperature increase near the liner surface during the time ~τ will be ~2000 K for the filamentary magnetic field of 100 T. To match the pitch angle to that inferred from the analysis of the experimental results presented in Ref. 1, one has to have the azimuthal field ~250 T, i.e., axial current (flowing predominantly on the liner surface) ~ 4.5 MA.

A similar scenario of the magnetic field compression could be realized if the plasma fills the whole volume between the electrodes early in the pulse. This plasma would be swept by the pinch force towards the liner surface, with the axial magnetic field swept together with the plasma. At the time-scale of interest for the experiments of the type [1], the plasma can be considered perfectly conducting, and the flux will be frozen into it. The presence of conducting surfaces limiting the gap from the top and the bottom, would again cause the distortion of the poloidal field lines similarly to what is shown in Fig.1.

To address the problem of the origin of an early plasma in the gap, we discuss the conditions in this gap assuming that there is no plasma there at all, just perfect vacuum. Then, for the geometrical parameters mentioned above, we find that the axial electric field at early times is of order of 300 kV/cm (it depends on the location). The ion with the corresponding high energy would cross the gap within one to a few nanoseconds, depending on the charge-to-mass ratio. There is an azimuthal field that could provide magnetic insulation, but, even prior to a significant compression of the bias field, it allows the ions to reach the opposite electrode moving along the helical field lines. The transit time would then contain a large multiplier B_φ / B_z, but for the early stages of the discharge this factor is only ~ 5-10, thereby allowing the ions to cross the gap in ~10 ns. Copious electrons produced by the 300 keV ion when it hits the cathode travel in the opposite direction with near-relativistic speed and reach the opposite electrode almost instantaneously, thereby creating a positive feedback for the avalanche development. This opens up a possibility for the vacuum breakdown of the gap, although it is impossible now to produce a reliable picture (which will depend, among other things, on the materials used and the cleanliness of the surfaces). This mechanism may operate more efficiently near the gap that connects the diode with the power transmission line. One may expect some changes in the inductance of the load early in the pulse, caused by the appearance of an additional current channel. Direct measurements of the magnitude and direction of the magnetic field early in the current pulse – and even the composition of the swept-in plasma – may be possible using streaked visible spectroscopy [8].

Later in the pulse the axial field, even if enhanced to 1-2 MG, will be completely overwhelmed by the toroidal field, and the implosion will proceed in a "standard" Z-pinch way. However, the imprint of the early helical perturbations will affect the initial composition of the modes growing during the further implosion, very much as described by Awe et al. [1].

In summary: Effect of compression of a pre-imposed axial magnetic field by an axial current in a moderate-density plasma has been assessed. Assuming that plasma with the electron density ~10^{17} cm^{-3} is formed early in the pulse, we come to a conclusion that the axial field near the liner surface can be enhanced by a factor of ~ 10 compared to its initial value.

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